

# Alphabets, strings and formal languages

## Lecture 6

# Symbols

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- There are symbols that do not take values. Example: the letter A (as a letter not as a variable, of course)
- A rock (on its own) is not a symbol

# Concatenation and strings

Definition: Let  $A$  be a set of symbols. A **concatenation of symbols** of  $A$  is the operation of **joining** together **an n-tuple of symbols of  $A$ , preserving the order of the symbols in the n-tuple.**

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Definition: The object **generated** by the concatenation of an  $n$ -tuple of symbols in  $A$  is called **string (of length  $n$ ) over  $A$**

# Example of string generation

Thus, in order to generate a string  $s$  of length  $n$  over a given set  $A$ , select first an  **$n$ -tuple**

$$(a_1, a_2, \dots, a_n) \in A \times A \times \dots \times A$$

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– Example: Given  $A = \{1, a, b\}$

$$\underbrace{(b, a, 1, b)}_{\substack{\text{Select an } n\text{-tuple} \\ (n=4 \text{ in this case})}} \longrightarrow \underbrace{s = ba1b}_{\text{generate the string}}$$

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$$\underbrace{(b, a, 1, b)}$$

Select an  $n$ -tuple

( $n=4$  in this case)



$$\underbrace{s = ba1b}$$

generate the string

**This is a good model for the operation of writing**

# Properties of strings

Strings inherit all the properties of the tuples.

In particular,

- The order of the symbols in the string is important
- The number of symbols in a string is important

Definition: Two **strings are different** if and only if **the tuples** used to write them **are different**

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3.  $\lambda\lambda = \lambda$

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- A **picture** stores an image as an array of strings of pixels

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As an operation **reading** is (somewhat) the **inverse** of **writing** (or **concatenation**)

- **Reading** is **analysis** (identification of components) while **concatenation** is **synthesis** (object composition)

# Illustration

Consider the string over  $\{x, 1, \Omega\}$   $s = 1x11x$ .

The reading of  $s$  is, step by step:

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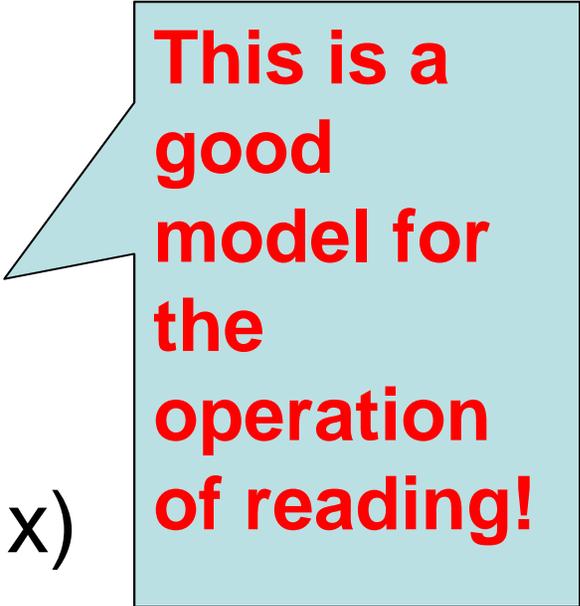
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2.  $1\underline{x}11x \rightarrow (1, x)$
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4.  $1x1\underline{1}x \rightarrow (1, x, 1, 1)$
5.  $1x11\underline{x} \rightarrow (1, x, 1, 1, x)$
6. Halt: string is read!



**This is a  
good  
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# Are write and read functions?

Let's put our models in mathematical context:

$$\textit{Write} : \bigcup_{j=1}^{\infty} \left( \prod_{i=1}^j \mathbf{A} \right) \times \bigcup_{j=1}^{\infty} \mathbf{A}^j$$

$$\textit{Write}(a_1, a_2, \dots, a_j) = \{a_1 a_2 \dots a_j\}$$

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**Write is a function** since a  $j$ -tuple over  $A$  never generate more than one string!

# What about read?

Here is the model:

$$\text{Read} : \bigcup_{j=1}^{\infty} A^j \times \bigcup_{j=1}^{\infty} \bigcup_{i=1}^j (X A)$$

$$\text{Read}(a_1 a_2 \cdots a_j) = \{(a_1, a_2, \cdots, a_j)\}$$

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$$\text{Read} : \bigcup_{j=1}^{\infty} A^j \times \bigcup_{j=1}^{\infty} \prod_{i=1}^j A$$

This is not a  
general  
description!

$$\text{Read}(a_1 a_2 \cdots a_j) = \{(a_1, a_2, \cdots, a_j)\}$$

This relation **is not**, in general, **a function**

# Read is not a function

Counterexample:

Let  $A = \{1, a, b, b1\}$ . Consider the string over  
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**Not a singleton: not a function!!**

# Alphabets

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The **reading** of a string depends on **the set of all symbols available for writing**. Some sets render more than one reading

Definition: A finite, nonempty set is an **alphabet** if each string over it renders a unique reading

# Write is one – to – one

Theorem: Over an alphabet, Write is one –  
to – one

Proof:

Let  $(a_1, a_2, \dots, a_j) \neq (b_1, b_2, \dots, b_j)$

Then, there is  $i, 1 \leq j \leq l$  so that  $a_i \neq b_i$ ,

Therefore, by definition of equality of  
strings:

$\text{Write}(a_1, a_2, \dots, a_j) \neq \text{Write}(b_1, b_2, \dots, b_j)$

# An important corollary

Corollary: Over an alphabet, Read is a function and it is in fact, the inverse of Write

Proof: Since Write is one to one, for each string  $s$  there is a unique tuple  $t$  such that  $\text{Write}(t) = s$ . Therefore,  $\text{Read}(s) = \{t\}$ , is a singleton. Also,  $\text{Read}(\text{Write}(t)) = t$  and  $\text{Write}(\text{Read}(s)) = s$ . Therefore, Read is the inverse of Write.

# Examples

## Alphabets:

$A = \{a, b, c\}, B = \{0, 1\}$

$C = \{ab, ac, bc\}, D = \{00, 01\}$

This is called  
binary alphabet

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$$C = \{ab, ac, bc\}, D = \{00, 01\}$$

## Non-alphabet:

$$E = \{a, b, ab\}$$

$$A \cup C = \{a, b, c, ab, ac, bc\}$$

**Remark:** As the last example shows: the union of two alphabets is not always an alphabet!!!  
(**Despite what people may say!!!**)

# Are these artificial examples?

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It is worth pointing out that, in general, proving that a set is indeed an alphabet may be difficult: not only sets that contain strings produce more than one reading!

# Some biochemical alphabets

- **DNA alphabet:**
  - $D = \{A, T, G, C\}$
- **RNA alphabet:**
  - $R = \{A, U, G, C\}$

- **Protein transcription alphabet:** symbols are “codons”. Each codon is formed with three RNA symbols →

		Second letter				
		U	C	A	G	
First letter	U	UUU UUC UUA UUG Phenyl-alanine Leucine	UCU UCC UCA UCG Serine	UAU UAC UAA UAG Tyrosine Stop codon Stop codon	UGU UGC UGA UGG Cysteine Stop codon Tryptophan	U C A G
	C	CUU CUC CUA CUG Leucine	CCU CCC CCA CCG Proline	CAU CAC CAA CAG Histidine Glutamine	CGU CGC CGA CGG Arginine	U C A G
A	AUU AUC AUA AUG Isoleucine Methionine; start codon	ACU ACC ACA ACG Threonine	AAU AAC AAA AAG Asparagine Lysine	AGU AGC AGA AGG Serine Arginine	U C A G	
G	GUU GUC GUA GUG Valine	GCU GCC GCA GCG Alanine	GAU GAC GAA GAG Aspartic acid Glutamic acid	GGU GGC GGA GGG Glycine	U C A G	

# Some biochemical alphabets

- **DNA alphabet:**
  - $D = \{A, T, G, C\}$
- **RNA alphabet:**
  - $R = \{A, U, G, C\}$

Here is an example of a natural alphabet that contains strings!

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		Second letter				
		U	C	A	G	
U	UUU UUC	UCU UCC UCA UCG	UAU UAC	UGU UGC	UAA UAG	UGA UGG
	Phenyl-alanine					
C	CUU CUC CUA CUG	CCU CCC CCA CCG	CAU CAC	CGU CGC CGA CGG	CAA CAG	UCA UAG
	Leucine					
A	AUU AUC AUA	ACU ACC ACA ACG	AAU AAC	AGU AGC	AAA AAG	AUA AUG
	Isoleucine					
G	GUU GUC GUA GUG	GCU GCC GCA GCG	GAU GAC	GGU GGC GGA GGG	GAA GAG	GUA GUG
	Valine					

# Languages

Definition: A **language** or, more specifically, a **formal language over an alphabet**  $A$  is a set of strings over  $A$  or the empty set

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Examples:

1. Every alphabet is a formal language over itself
2. The set of natural numbers is a language over the alphabet  $A=\{0, 1, 2, \dots, 9\}$

# More examples of languages

- Let  $A = \{0, 1\}$  then  $A$  is a language over  $A$
- Languages over the binary alphabet  $\{0, 1\}$  are pervasive in computation
- $A = \{\lambda\}$  is a language (and an alphabet)
- Natural languages are formal languages that in addition to **denotation** have **connotation** (intended meaning)

# Basic facts about languages

## Theorem 5.3:

- (a) The union of two languages over the same alphabet is a language
- (b) The intersection of two languages over the same alphabet is a language
- (c) The difference of two languages over the same alphabet is a language

Proof of (a)

Let  $L$  and  $M$  be languages over the same alphabet  $A$ .

Then,  $(\forall s)(s \in L \cup M) \Rightarrow s \in L \vee s \in M$

$\Rightarrow s$  is a string over  $A$

Thus,  $L \cup M$  is a language over  $A$ .

Proof of (b)

Let  $L$  and  $M$  be languages over the same alphabet  $A$ .

Then,  $(\forall s)(s \in L \cap M) \Rightarrow s \in L \wedge s \in M$

$\Rightarrow s$  is a string over  $A$

Thus,  $L \cap M$  is a language over  $A$ .

Proof of (c)

Let  $L$  and  $M$  be languages,  $L \subseteq M$ .

Then,  $(\forall s)(s \in M) s$  is a string over  $A$

Since  $M - L \subseteq M$

$(\forall s)(s \in M - L \Rightarrow s \in M)$

Therefore,  $(\forall s)(s \in M - L$  is a string over  $A)$

# String concatenation

Definition: Let  $L$  and  $S$  be nonempty languages. The **string concatenation over  $L$**  is defined to be the operation

$$Cat(s, t) = st$$

We define  $Cat(L) = \{st : s \in L \wedge t \in L\}$

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Example: Let  $L$  be the language of **all strings over the alphabet  $A = \{a, b, 1\}$** . Let's take  $s = **abaab**$  and  $t = **1a1ab1**$  (two strings in  $L$ ). Their concatenation is

$$Cat(s, t) = st = **abaab1a1ab1**$$

# Properties of concatenation

**Associativity**  $(\forall s, t, r)(s, t, r \text{ strings over } A)$

$$\text{Cat}(s, \text{Cat}(t, r)) = \text{Cat}(\text{Cat}(s, t), r)$$

**Non-commutativity**

$(\exists s, t)(s, t \text{ strings over } A) \wedge \text{Cat}(s, t) \neq \text{Cat}(t, s)$

**Existence of an identity (null string)**

$(\exists \lambda)(\lambda \text{ string}) \wedge ((\forall s)(s \text{ string over } A))$

$$\text{Cat}(\lambda, s) = \text{Cat}(s, \lambda) = s$$

Observation: the null string **is not** the empty set

# Closure under string concatenation

Definition: A language  $L$  is said to be **closed under string concatenation** if  $L = \text{Cat}(L)$

Examples:

- The language  $L = \{\lambda\}$  is closed under string concatenation
- The language  $L = \emptyset$  is also closed under string concatenation (Why?)
- The language  $L$  of all strings over an alphabet is closed under string concatenation

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# More examples

The language  $L = \{0, 00, 000, \dots\} = \{0^n : n \geq 1\}$  **is closed under concatenation**. In order to see this, take any two strings in  $L$ , this is  $s = 0^n$  and  $t = 0^m$ ,  $s, t \geq 1$ .

Then,  $st = 0^n 0^m = 0^{n+m} \in L$

The language  $L = \{01^n : n \geq 1\}$  **is not closed under string concatenation** since, for instance:  $Cat(01, 011) = 01011 \notin L$

# Why does closure matters?

String concatenation is pervasive as a language operation. If a language is closed under string concatenation, the operation of concatenation is a **language invariant** (i.e. does not alter the language). In particular, if the language is closed under string concatenation, any concatenation satisfies the predicate that defines membership in the language.

Example:

$Q(w) : "w \text{ string over } \{0, 1\} \wedge w \text{ ends with } 0"$

$L = \{w : Q(w) = 1\}$

Now, since the concatenation of any finite collection of strings over  $\{0,1\}$  ending with 0 is a string that ends with 0,  $L$  is closed under string concatenation. Therefore,

$(\forall s, w \in L) Q(sw) = 1$

# Theorem on string concatenation

Theorem 5.4: If  $L$  is a nonempty language that is closed under string concatenation, then  $L$  is finite if and only if  $L = \{\lambda\}$ .

Proof: If  $L = \{\lambda\}$ , since  $Cat(\lambda, \lambda) = \lambda\lambda = \lambda$  we have that  $Cat(L) = \{\lambda\} = L$ . Therefore,  $L$  is closed under string concatenation, and finite.

If  $L \neq \{\lambda\}$ , then there exists  $s \in L \wedge s \neq \lambda$ .

# Proof (cont.)

Since  $L$  is closed under string concatenation the following inductive construction produces a infinite subset of  $L$

Base case:

$$\text{Cat}(s, s) = ss = s^2 \in L \quad (\text{true because } L \text{ is closed})$$

Induction:

$$(\forall n) \text{Cat}(s, s^{n-1}) = s^n \in L \Rightarrow \text{Cat}(s, s^n) = s^{n+1} \in L$$

(true because  $L$  is closed)

Thus, the set  $\{s, s^2, \dots, s^n, \dots\} = \{s^n : n \geq 1\} \subseteq L$  is an infinite subset of  $L$ . And therefore,  $L$  is infinite.

# Closure of a language

Definition: Let  $A$  be an alphabet and  $L$  be a language over  $A$ . Then, the **star – closure** (or Klein – closure) of  $L$  is the smallest language over  $A$  that contains  $L$  and is closed under string concatenation. The star – closure of  $L$  is denoted  $L^*$ .

Remark: In general,  $L$  is a subset of  $L^*$ . In fact,  $L = L^*$  if and only if  $L$  is star – closed

# Star – closure of an alphabet

Theorem: Let  $A$  be an alphabet. Then, any language  $L$  over  $A$  is a subset of  $A^*$

Proof: Let  $s$  be a string in  $L$ . Since  $L$  is a language over  $A$ ,  $s$  is the result of a concatenation of symbols in  $A$ . But then,  $s$  is an element of  $A^*$

# Encodings

Let  $A, B$  be two alphabets. An **encoding of  $A$  in  $B$**  is a **one-to-one** mapping

$$e : A \rightarrow B^*$$

satisfying the **additional condition** that the set

$$e(A) = \{w : w \in B^* \wedge (\exists x \in A) \wedge e(x) = w\}$$

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Example: Let  $A = \{0, 1\}; B = \{a, b\}$ . Then,

$e(0) = a; e(1) = b$  is an encoding, but

$e(0) = ab; e(1) = abab$ , is NOT

# Popular examples of encodings

**ASCII:** The ASCII encoding specifies a one-to-one correspondence between an alphabet of letters and punctuation symbols of a written language and a set of binary strings (i.e. strings over  $\{0, 1\}$ )

Some values are:

$e(A)=1000001$

$e(B)=1000010$

$e(C)=1000011, \dots$  etc,

$e(,)=01011100$

$e(.)= 01011110, \dots$  etc.

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**Hexadecimal numbers:** the encoding specifies a one-to-one correspondence between the alphabet

$A=\{0, 1, 2, \dots, 9, A, B, C, D, E, F\}$

And strings over the alphabet

$B=\{0, 1, 2, \dots, 9\}$ . The encoding is:

$e(0)=0$

$e(1)=1, \dots, e(9)=9,$

$e(A)=10, e(B)=11, e(C)=12,$

$e(D)=13, e(E)=14, e(F)=15$

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Encodings are **alphabet translators**. Their existence ensures that **the ability to represent objects does not depend on the choice of an alphabet**

Indeed, if  $A, B$  are two different alphabets, and  $w = a_1 a_2 \dots a_n$  is a string over  $A$ . Then, its “translation” as a string over  $B$  is

$$e(a_1)e(a_2)\dots e(a_n)$$

# A simple illustration

The word in Spanish language:

**CABA**

is encoded by ASCII over  $\{0, 1\}^*$  as

$$\begin{aligned} & \mathbf{e(C)e(A)e(B)e(A)} \\ & \mathbf{= 1000011100000110000101000001} \end{aligned}$$